

Combining Modern Codes and Set-Partitioning for Multilevel Storage Systems

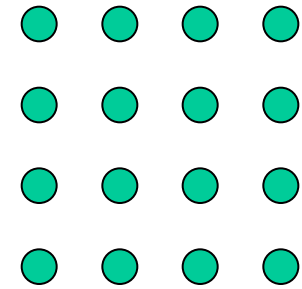
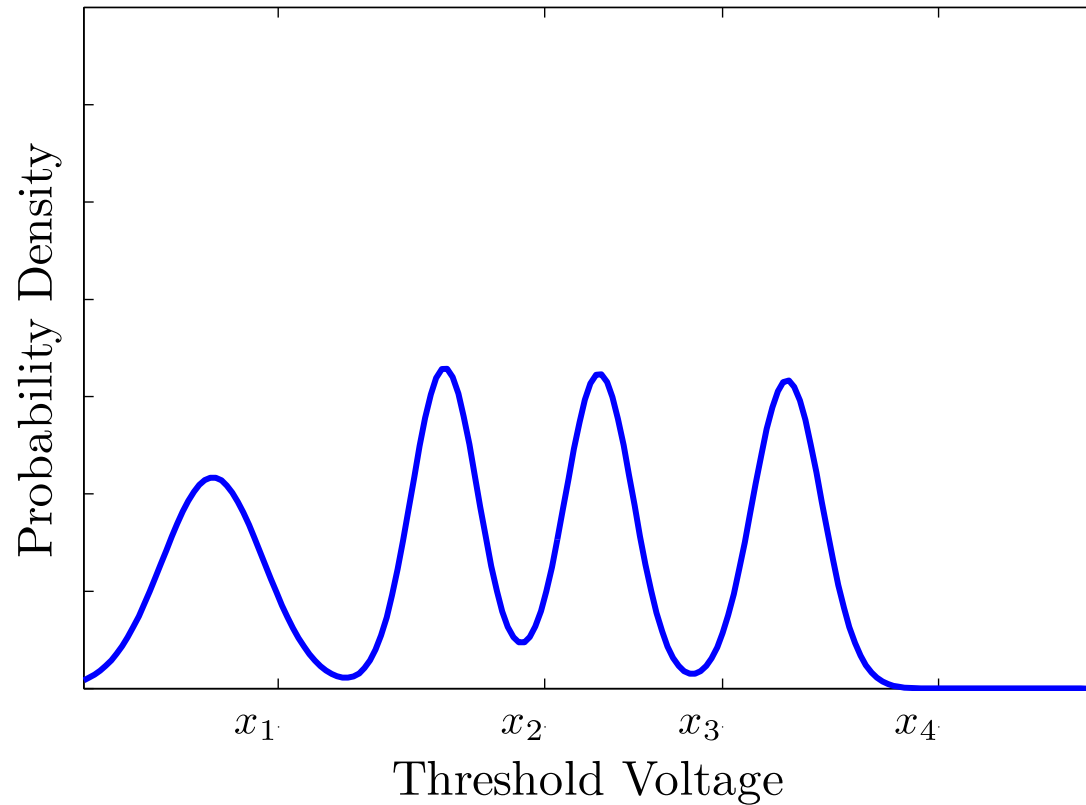
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Multilevel Flash is Essentially PAM

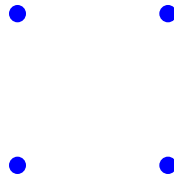
(c) $f(x + n_p + n_w + n_r)$



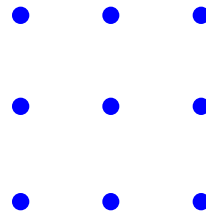
16-QAM

Constellation Size

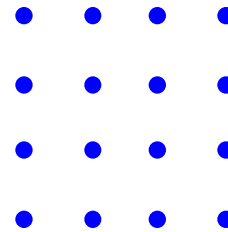
4-QAM



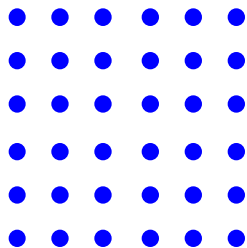
9-QAM



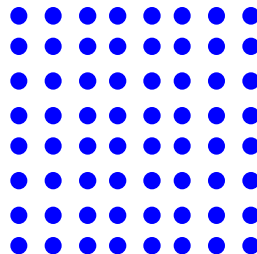
16-QAM



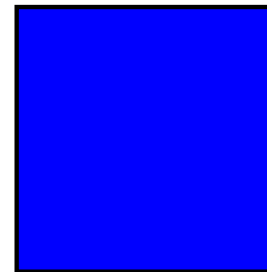
36-QAM



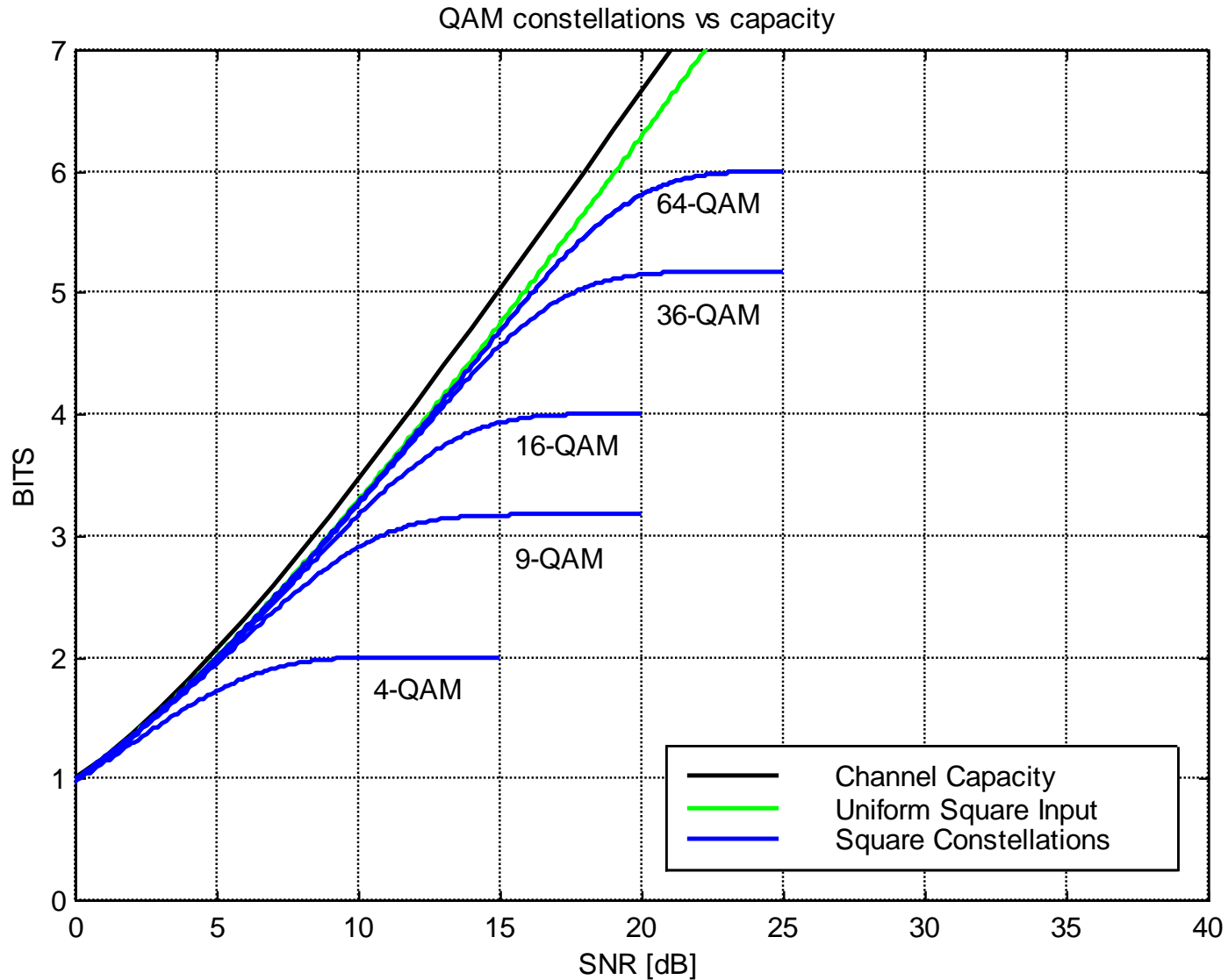
64-QAM



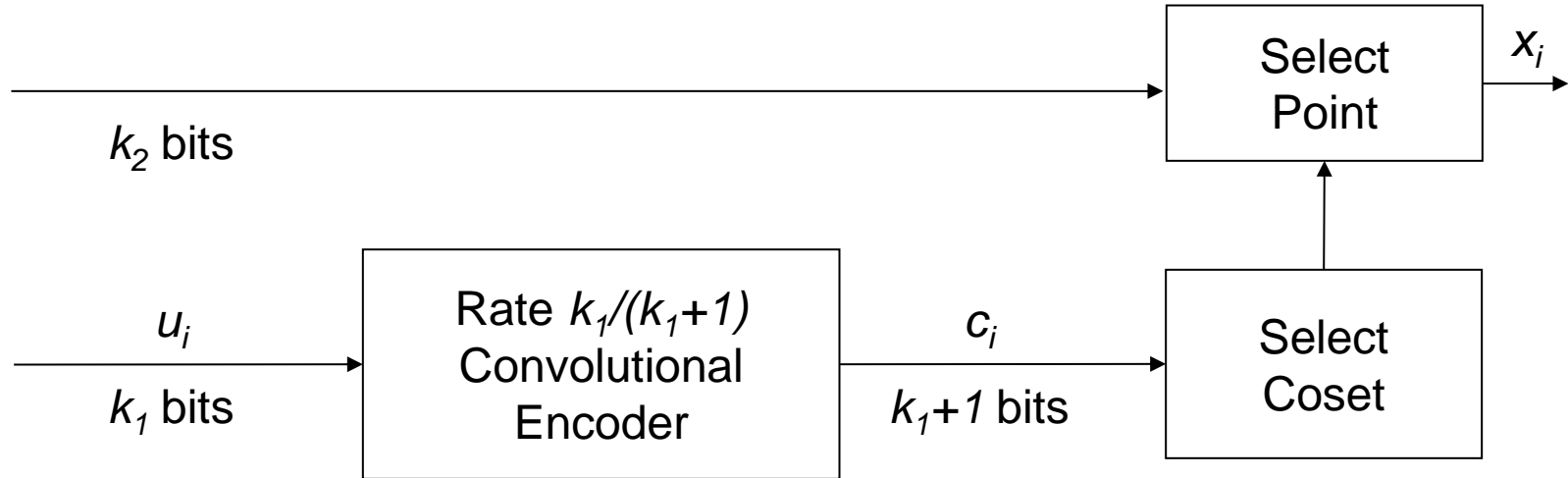
Uniform Square



Maximum Mutual Information



Set Partitioning Encoder*[1]



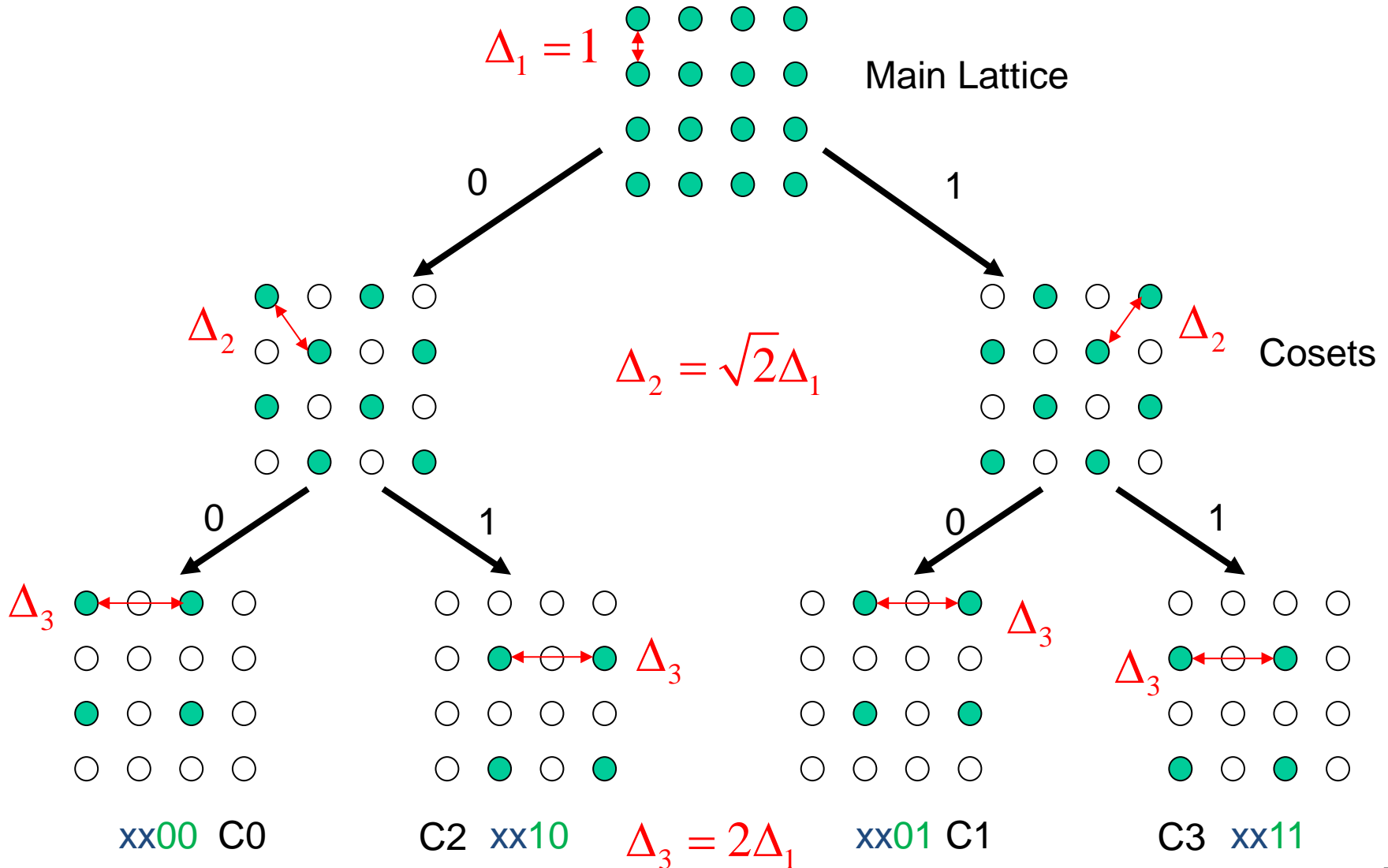
* - Also became to be known as Trellis Coded Modulation (TCM)

[1] G. Ungerboeck, "Channel coding with multilevel/phase signals," *IEEE Trans. Inf. Theory*, vol. 28, no. 1, pp. 55–67, Jan. 1982.

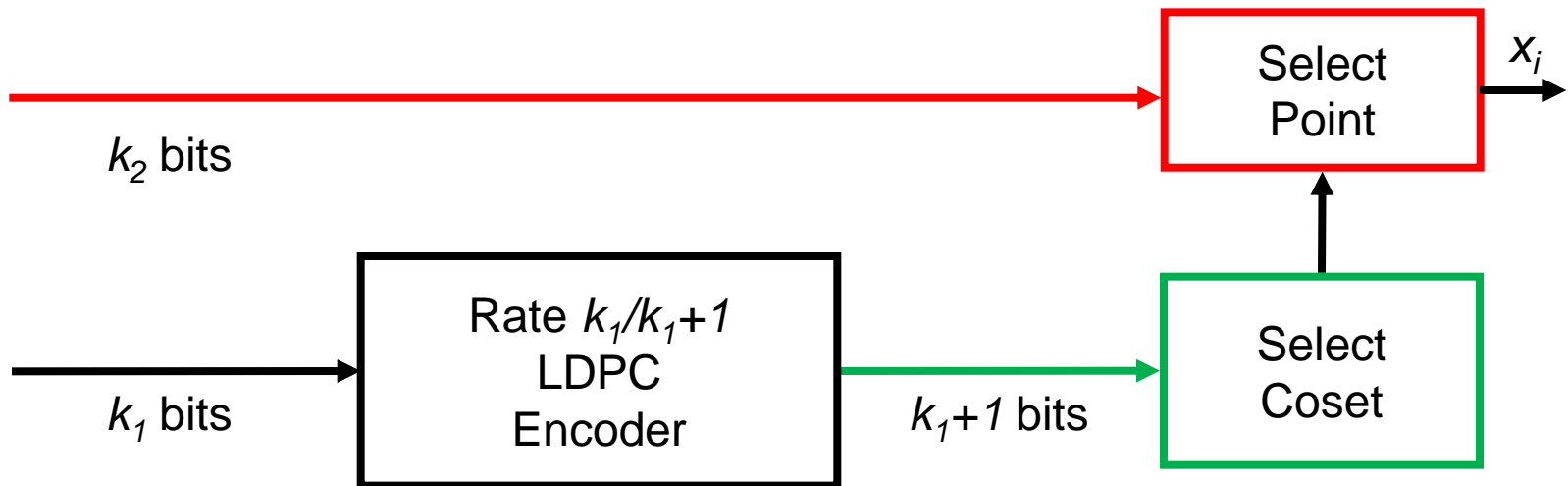
Divide and Conquer

- Constellation points are labeled to guarantee large distance between points that differ only in uncoded bits.
- Follow simple rules to design coset code maximizing the Minimum Squared Euclidean Distance (MSED).

Mapping by Set Partitioning



Coset and Point selection



Uncoded Bits

- Bit Error Rate (BER)
 - Union bound for M-QAM square constellation (assuming Gray encoding)
 - k level set-partitioned

$$d_{\min} = 2^{k/2} A$$

$$BER \leq \frac{2^{\log_2 M - k} - 1}{\log_2 M - k} * Q\left(\frac{d_{\min}}{2\sigma}\right)$$

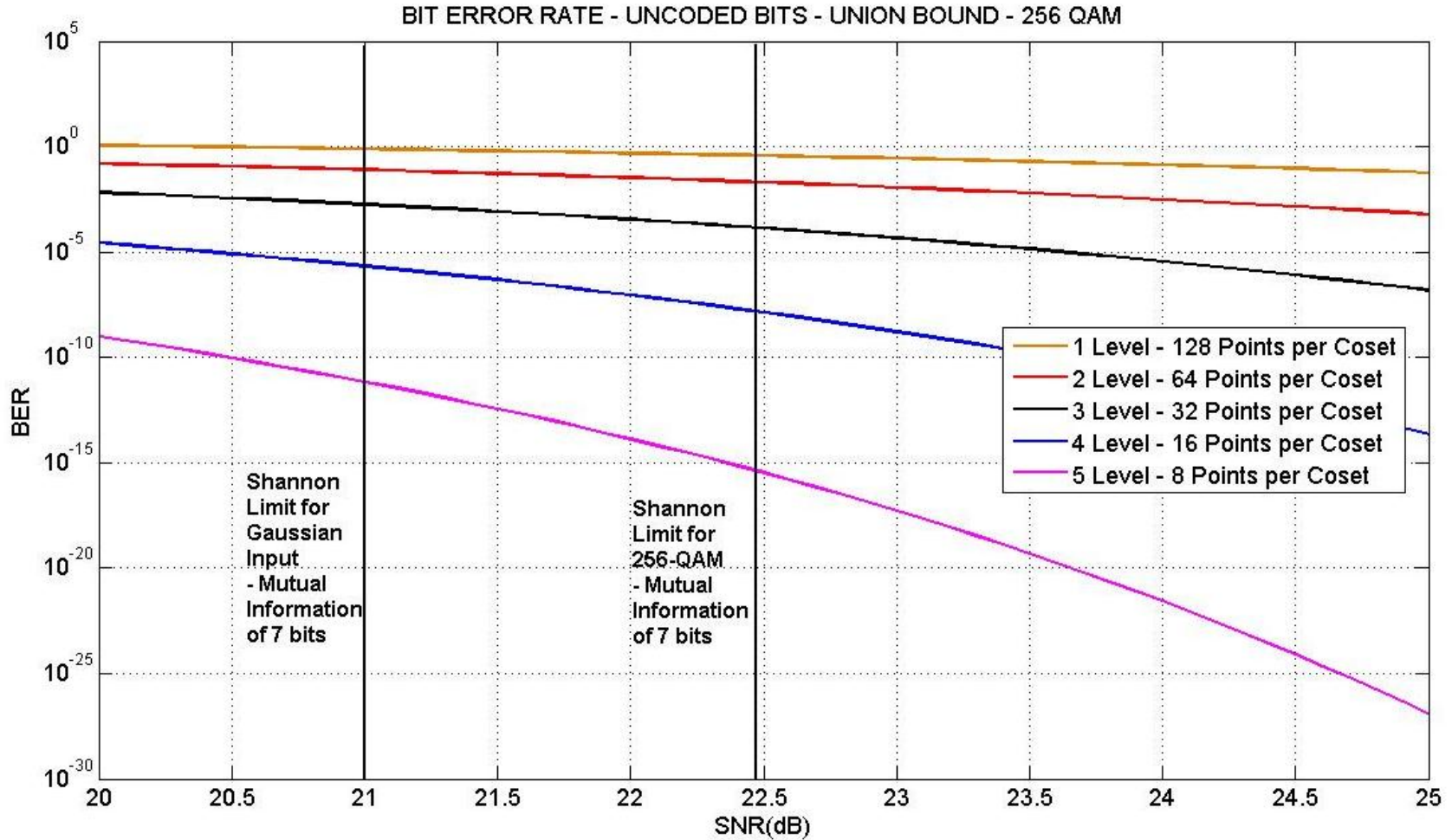
$$A = \sqrt{\frac{E_{s,avg}}{E_{s,avg}^-}}$$

where,

- A is the minimum distance between two constellation points,
- d_{\min} is the minimum intra-coset distance.

Shannon Limit, Target

- For 256-QAM signaling,



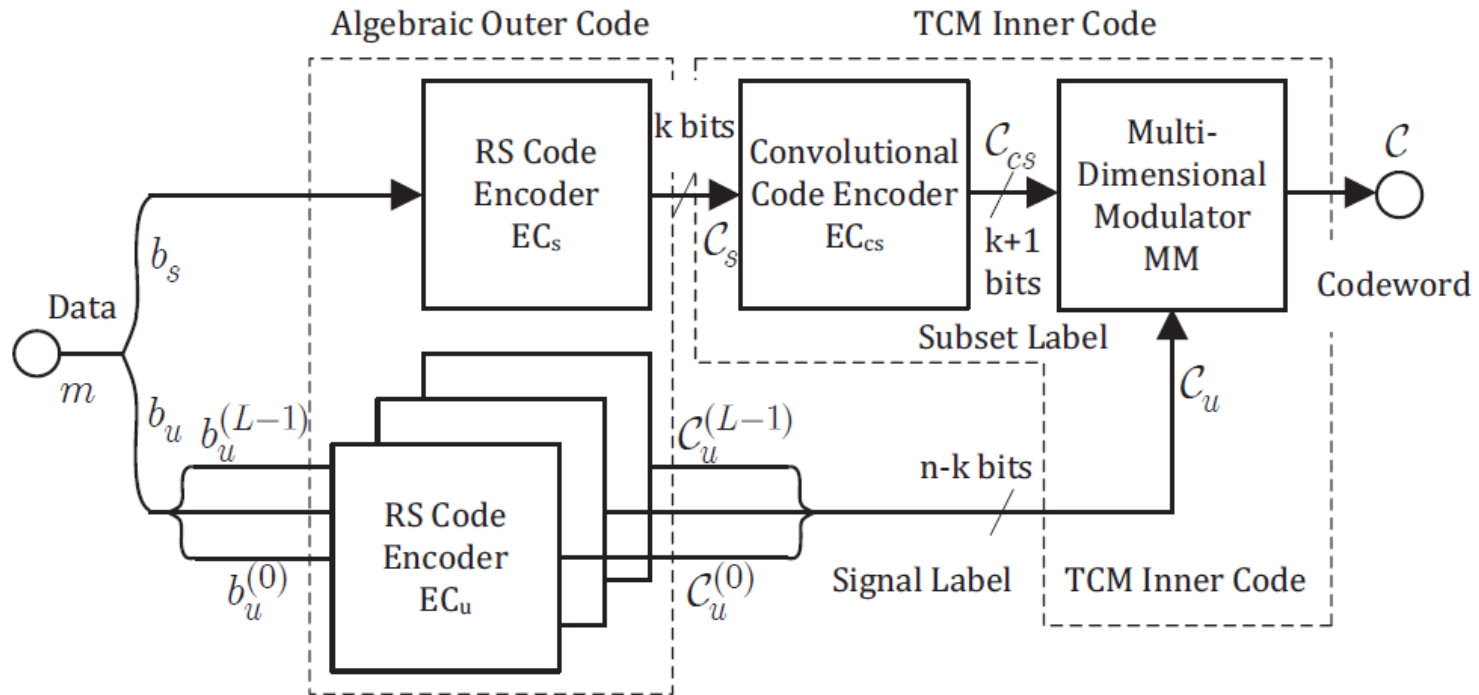
- Existing work:
 - Set-partitioned TCM with outer codes are empirically better than BCH Codes[2]-[4].
 - Lower complexity compared to using just BCH codes for a required level of error correction.
 - In concatenation with multi-level outer block codes, achieve target Word error rate (WER)[2].

[2] Oh, Jieun; Ha, Jeongseok; Moon, Jaekyun; Ungerboeck, Gottfried, "RS-Enhanced TCM for Multilevel Flash Memories," *IEEE Transactions on Communications*, vol.61, no.5, pp.1674-1683, May 2013.

[3] Shu Li; Tong Zhang, "Improving Multi-Level NAND Flash Memory Storage Reliability Using Concatenated BCH-TCM Coding," *IEEE Transactions on Very Large Scale Integration (VLSI) Systems*, vol.18, no.10, pp.1412-1420, Oct. 2010.

[4] F. Sun, S. Devarajan, K. Rose, and T. Zhang, "Multilevel flash memory on-chip error correction based on trellis coded modulation," in *Proc. IEEE Int. Symp. Circuits Syst. (ISCAS)*, pp. 1443–1446, May 2006,.

Set-Partition Coding for Multi-Level Flash – Example[2, Fig. 1]



[2] Oh, Jieun; Ha, Jeongseok; Moon, Jaekyun; Ungerboeck, Gottfried, "RS-Enhanced TCM for Multilevel Flash Memories," *IEEE Transactions on Communications*, vol.61, no.5, pp.1674-1683, May 2013.

Set-Partition Coding for Multi-Level Flash – Modern Codes

- With Modern Codes:
 - Modern codes have not been used with Set-Partitioning for Flash so far, but they have been proposed[5].
 - LDPC based systems with Set-Partitioning for DSL[6][7] have been proposed.

[5] A. Ramamoorthy, A. Wu, and P. Sutardja, "Method and system for error correction in flash memory," US Patent 7844879B2, Nov. 2010.

[6] Eleftheriou, E.; Ölçer, S.; Sadjadpour, H., "Application of capacity approaching coding techniques to digital subscriber lines," *IEEE Communications Magazine*, vol.42, no.4, pp.88-94, Apr 2004.

[7] Eleftheriou, E.; Olcer, S., "Low-density parity-check codes for digital subscriber lines," in *Proc. IEEE International Conference on Communications*, vol.3, no., pp.1752-1757 vol.3, 2002.

System Design Guidelines

- Multidimensional constellations
 - Extension of 2-D constellation labeling[2][3].
- Uncoded Bits
 - Can be protected by Multi-level coding further if required[8].

[2] Oh, Jieun; Ha, Jeongseok; Moon, Jaekyun; Ungerboeck, Gottfried, "RS-Enhanced TCM for Multilevel Flash Memories," *IEEE Transactions on Communications*, vol.61, no.5, pp.1674-1683, May 2013.

[3] Shu Li; Tong Zhang, "Improving Multi-Level NAND Flash Memory Storage Reliability Using Concatenated BCH-TCM Coding," *IEEE Transactions on Very Large Scale Integration (VLSI) Systems*, vol.18, no.10, pp.1412-1420, Oct. 2010.

[8] Wachsmann, U.; Fischer, R. F H; Huber, J.B., "Multilevel codes: theoretical concepts and practical design rules," *IEEE Transactions on Information Theory*, vol.45, no.5, pp.1361-1391, Jul 1999.

Non-Binary LDPC

- NB-LDPC has its advantages
 - Inherently a perfect fit for a Multi-level Flash cells
 - Decoding advantage from symbol reliabilities.
 - Coset code with required performance is achievable !
 - Lower rate coset code than usual (can get rid of error floors).
 - Complexity of decoding reduces as field size goes down for Non-Binary LDPC codes.
- Modern codes are more powerful.

Research Plan

- Analyzing the system requirements so as to
 - Characterize the number of dimensions to signal.
 - Set-Partition a constellation to required levels.
 - Designing a low rate coset code to meet the target BER.
- Theoretical research scope
 - Study of Non-Binary codes over q-ary signaling
 - Characterization of Non-Binary codes e.g. Heuristic algorithms to find out the nearest neighbor characteristics.
 - Possible code design and decoding guidelines for specific applications.
- Issues
 - Read precision
 - LDPC codes perform better with more quantization levels.
 - Complexity vs. Performance trade-off.