Iterative Decoders on Noisy Hardware

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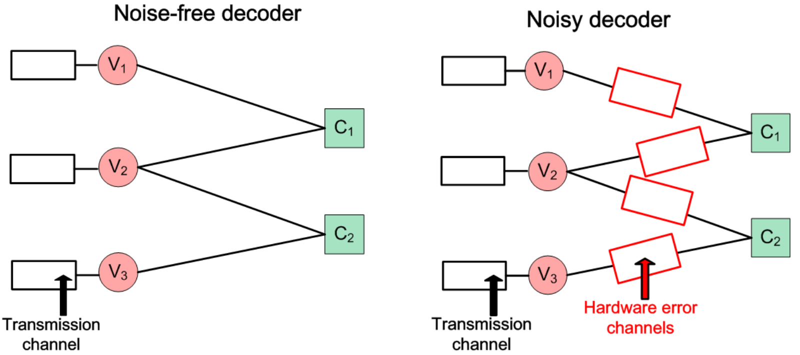
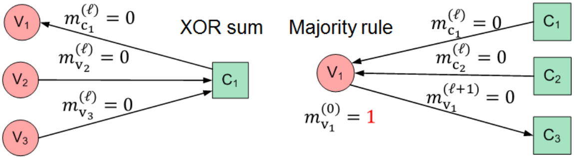
Noisy Hardware Problem

- Upcoming nano-scale devices will be error-prone.
  - P. Gupta et al., "Underdesigned and Opportunistic Computing in Presence of Hardware Variability," IEEE Trans. Comput.-Aided Design Integr. Circuit Syst., 2013
- Information processing algorithms will be implemented on unreliable devices.
- Characterization of the effects of noisy hardware becomes an important task.
- We derive the error rates of iterative message passing decoders under both transmission and hardware errors.

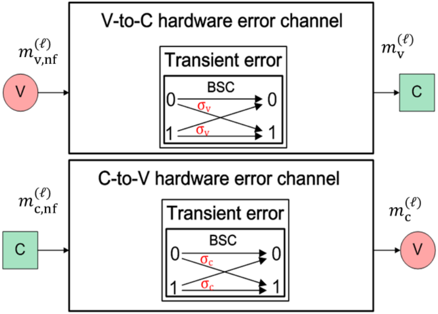

Noise-free decoder
V1
V2
V3
C1
C2
Transmission channel
Noisy decoder
V1
V2
V3
C1
C2
Transmission channel
Hardware error channels

Transmission Error Correction in Nominal Gallager B Decoder



- $p^{(\ell)}$: error probability of variable node messages at iteration ℓ.
- $q^{(\ell)}$: error probability of check node messages at iteration ℓ.
- We know error propagation functions ϕ_{nf} and ψ_{nf} such that $p^{(\ell+1)} = \phi_{\mathrm{nf}}(q^{(\ell)})$ and $q^{(\ell)} = \psi_{\mathrm{nf}}(p^{(\ell)})$.
- The recursive expression for the bit error rate as a function of iteration number ℓ is then $p^{(\ell+1)} = \phi_{\mathrm{nf}}(\psi_{\mathrm{nf}}(p^{(\ell)}))$.

Hardware Error Model



- Our noisy decoder model has hypothetical noise-free nodes and hardware error channels.
- Binary symmetric channel (BSC) is placed between the hypothetical noise-free node output $m_{\mathrm{v,nf}}^{(\ell)}$ ($m_{\mathrm{c,nf}}^{(\ell)}$) and the real output $m_{\mathrm{v}}^{(\ell)}$ ($m_{\mathrm{c}}^{(\ell)}$) to model the effects of the transient errors.

Gallager B Decoder Under Transmission and Transient Errors

- For a (d_v, d_c)-regular LDPC code, the recursive expression for the bit error rate as a function of iteration number ℓ is

$$\boxed{p^{(\ell+1)} = (1-2\sigma_v)\phi_{\mathrm{nf}}\Big((1-2\sigma_c)\psi_{\mathrm{nf}}(p^{(\ell)}) + \sigma_c\Big) + \sigma_v,}$$

  where ϕ_{nf} and ψ_{nf} are Gallager B decoder error propagation functions for variable and check nodes, respectively.

- When ϵ, σ_v, and σ_c are small, the residual error rate is

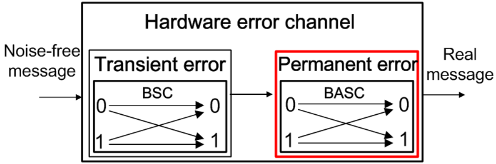
$$\boxed{p^{(\ell)} \approx \begin{cases} \sigma_v & d_v > 3, \\ \frac{\sigma_v + 2\epsilon\sigma_c}{1-2(d_c-1)\epsilon} & d_v = 3, \end{cases}}$$

  for $\ell \to \infty$. Here, ϵ is the error probability of the channel output.

Gallager B Decoder Under Transmission, Transient and Permanent Errors

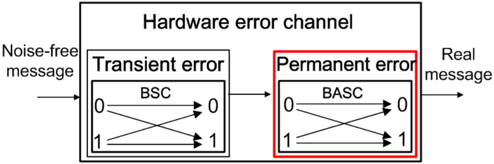
- Permanent errors: memory cells stuck at certain values.

Gallager B Decoder Under Transmission, Transient and Permanent Errors



- Permanent errors: memory cells stuck at certain values.
- Number of stuck-at-1 cells $\neq$ number of stuck-at-0 cells in general.
- Binary asymmetric channel (BASC) between the transient error BSC channel output and the real message.

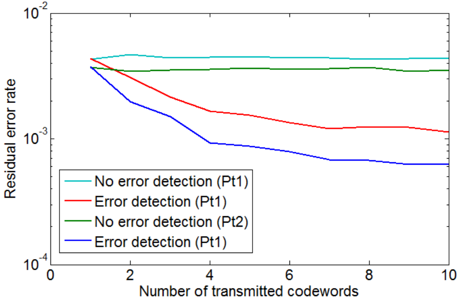
Gallager B Decoder Under Transmission, Transient and Permanent Errors



- Permanent errors: memory cells stuck at certain values.
- Number of stuck-at-1 cells $\neq$ number of stuck-at-0 cells in general.
- Binary asymmetric channel (BASC) between the transient error BSC channel output and the real message.
- Permanent and transient errors in variable node outputs dominate the residual error rate of the noisy decoder.

Reducing Residual Error Rate by Permanent Error Detection

- To reduce the residual error rate, we eliminate permanent errors through permanent error detection.
- Permanent errors in variable nodes are detected from carefully-chosen combinations of variable node messages.
- We ran simulations for a (3,6)-regular LDPC code of codelength 2640 on a noisy decoder.



Transient error	Pt1=0.001
Transient error	Pt2=0.0005
Stuck-at-1 error	0.003
Stuck-at-0 error	0.002
Transmission error	0.005

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Summary and Ongoing Work

- We mathematically characterized the error rates of noisy iterative message passing decoders.
- Analysis allows identification of crucial error and code parameters that impact the decoder output error rate.
  - E.g., the decoder output error rate is dominated by errors in the variable node output messages.
- We proposed and analyzed schemes that effectively suppress the impact of hardware errors on decoder output.
- We are also looking for codes robust to hardware errors.